Tutorial on the Use of Significant Figures

All measurements are approximations—no measuring device can give perfect measurements without experimental uncertainty. By convention, a mass measured to 13.2 g is said to have an absolute uncertainty of plus or minus 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit—it could be a "2"; then again, it could be a "1" or a "3". A mass of 13.20 g indicates an absolute uncertainty of plus or minus 0.01 g.

The objectives of this tutorial are:

—Explain the concept of signficant figures.
—Define rules for deciding the number of significant figures in a measured quantity.
—Explain the concept of an exact number.
—Define rules for determining the number of significant figures in a number calculated as a result of a mathematical operation.
—Explain rules for rounding numbers.
—Present guidelines for using a calculator.
—Provide some exercises to test your skill at significant figures.

Tutorial on the Use of Significant Figures, page 2

What is a "significant figure"?

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures.

Tutorial on the Use of Significant Figures, page 3

Rules for deciding the number of significant figures in a measured quantity:

(1) All nonzero digits are significant:

1.234 g has 4 significant figures,
1.2 g has 2 significant figures.

(2) Zeroes between nonzero digits are significant:

1002 kg has 4 significant figures,
3.07 mL has 3 significant figures.

(3) Leading zeros to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point:

0.001 ^{o}C has only 1 significant figure,
0.012 g has 2 significant figures.

(4) Trailing zeroes that are also to the right of a decimal point in a number are significant:

0.0230 mL has 3 significant figures,
0.20 g has 2 significant figures.

Tutorial on the Use of Significant Figures, page 3

(5) When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant:

> 190 miles may be 2 or 3 significant figures,
> 50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation. For example, depending on whether the number of significant figures is 3, 4, or 5, we would write 50,600 calories as:

> 5.06×10^4 calories (3 significant figures)
> 5.060×10^4 calories (4 significant figures), or
> 5.0600×10^4 calories (5 significant figures).

By writing a number in scientific notation, the number of significant figures is clearly indicated by the number of *numerical figures* in the 'digit' term as shown by these examples. This approach is a reasonable convention to follow.

U N I V E R S I T Y O F S O U T H C A R O L I N A

Stephen L. Morgan

Analytical Chemistry

Tutorial on the Use of Significant Figures, page 4

What is an "exact number"?

Some numbers are exact because they are known with complete certainty.

Most exact numbers are integers: exactly 12 inches are in a foot, there might be exactly 23 students in a class. Exact numbers are often found as conversion factors or as counts of objects.

Exact numbers can be considered to have an infinite number of significant figures. Thus, the number of apparent significant figures in any exact number can be ignored as a limiting factor in determining the number of significant figures in the result of a calculation.

Tutorial on the Use of Significant Figures, page 6

Rules for rounding off numbers

(1) If the digit to be dropped is greater than 5, the last retained digit is increased by one. For example,

12.6 is rounded to 13.

(2) If the digit to be dropped is less than 5, the last remaining digit is left as it is. For example,

12.4 is rounded to 12.

(3) If the digit to be dropped is 5, and if any digit following it is not zero, the last remaining digit is increased by one. For example,

12.51 is rounded to 13.

(4) If the digit to be dropped is 5 and is followed only by zeroes, the last remaining digit is increased by one if it is odd, but left as it is if even. For example,

11.5 is rounded to 12,
12.5 is rounded to 12.

This rule means that if the digit to be dropped is 5 followed only by zeroes, the result is always rounded to the even digit. The rationale for this rule is to avoid bias in rounding: half of the time we round up, half the time we round down.

Tutorial on the Use of Significant Figures, page 7

General guidelines for using calculators

When using a calculator, if you work the entirety of a long calculation without writing down any intermediate results, you may not be able to tell if an error is made. Further, even if you realize that one has occurred, you may not be able to tell where the error is.

In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately. For example,

$(5.00 / 1.235) + 3.000 + (6.35 / 4.0) = 4.04858... + 3.000 + 1.5875 = 8.630829...$

The first division should result in 3 significant figures. The last division should result in 2 significant figures. The three numbers added together should result in a number that is rounded off to the last common significant digit occurring furthest to the right; in this case, the final result should be rounded with 1 digit after the decimal. Thus, the correct rounded final result should be 8.6. This final result has been limited by the accuracy in the last division.

Most modern calcualtors allow you to carry all the results of intermediate calcuations in the display when performing a complex series of calcuations. By doing this, you can retain the results of each individual calculation step, and avoid having to re-enter intermediate results (a practice that may encourage rounding too soon). In this manner, you can completely avoid truncation errors introduced by rounding intermediate results

Warning: carrying all digits through to the final result before rounding is critical for many mathematical operations in statistics. Rounding intermediate results when calculating sums of squares can seriously compromise the accuracy of the result.

Tutorial on the Use of Significant Figures, page 8

Sample problems on significant figures

Instructions: print a copy of this page and work the problems. When you are ready to check your answers, go to the next page.

1. $37.76 + 3.907 + 226.4 = ?$

2. $319.15 - 32.614 = ?$

3. $104.630 + 27.08362 + 0.61 = ?$

4. $125 - 0.23 + 4.109 = ?$

5. $2.02 \times 2.5 = ?$

6. $600.0 / 5.2302 = ?$

7. $0.0032 \times 273 = ?$

8. $(5.5)^3 = ?$

9. $0.556 \times (40 - 32.5) = ?$

Tutorial on the Use of Significant Figures, page 9

Answer key to sample problems on significant figures

1. 37.76 + 3.907 + 226.4 = 268.1
2. 319.15 - 32.614 = 286.54
3. 104.630 + 27.08362 + 0.61 = 132.32
4. 125 - 0.23 + 4.109 = 129 (assuming that 125 has 3 significant figures).
5. 2.02 × 2.5 = 5.0
6. 600.0 / 5.2302 = 114.7
7. 0.0032 × 273 = 0.87
8. $(5.5)^3 = 1.7 \times 10^2$
9. 0.556 × (40 - 32.5) = 4

This answer assumes that 40 has two significant figures; however, that is not unambiguous, because there is no decimal point, and because it is not expressed in scientific notation. If 40 is an exact number (e.g., a count), then the result should be 4.17. If 40 has one signfiicant figure, then the answer is 4.

10. $45 \times 3.00 = 1.4 \times 10^2$
This answer assumes that 45 has two significant figures; however, that is not unambiguous, because there is no decimal point, and because it is not expressed in scientific notation. If 45 is an exact number (e.g., a count), then the result should be 1.35×10^2.

11. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715?
The average of these numbers is calculated to be 0.17118, which rounds to 0.1712 .

12. What is the standard deviation of the numbers in question 11?

The result that you get in calculating the standard deviation of these numbers depends on the number of digits retained in the intermediate digits of the calculation. For example, if you used 0.1712 instead of the more accurate 0.17118 as the mean in the standard deviation calculation, that would be wrong (don't round intermediate results or you will introduce propagated error into your calculations).

The Mathworks MATLAB std function gives 6.379655163094639e-004
Microsoft Excel gives 0.000637965516309464000000000000
A Hewlett-Packard 50G calculator gives 6.37965516309e-04

These results should be rounded to 0.0006380, which is 6.380×10^{-4} (expressed in scientific notation).

13. $3.00 \times 10^5 - 1.5 \times 10^2$ = ? (Give the exact numerical result, and then express that result to the correct number of significant figures).

U N I V E R S I T Y O F S O U T H C A R O L I N A

Stephen L. Morgan Analytical Chemistry

Links to other resources on the use of significant figures

Uncertainty in Measurements: A tutorial by Professor Frederick A. Sensese at Frostburg State University that shows how uncertainty arises from length, temperature, and volume measurements. How to count significant figures for a single measurement and for a series of measurements. How to round measurements to the correct number of significant figures. http://antoine.frostburg.edu/cgi-bin/senese/tutorials/sigfig/index.cgi

Significant figures quiz: A Javascript tutorial by Professor Frederick A. Sensese at Frostburg State University that lets you test your knowledge of the use of significant figures.
http://antoine.frostburg.edu/chem/senese/101/measurement/sigfig-quiz.shtml

Frequently asked questions about measurements, including FAQs on significant figures (such as:
"Why should the rules for propagating significant digits not be applied to averages?
"Why does 1101 cm - 1091 cm = 10 cm with 2 significant figures?"
"Are there simpler rules for counting significant digits?"
http://antoine.frostburg.edu/chem/senese/101/measurement/faq.shtml

Determining the number of significant figures: a drill by Scott Van Bramer of Widener University involving significant figures that presents you with succesive number displays and grades you on your answer to the question, "How many significnat figures are there?"
http://science.widener.edu/svb/tutorial/sigfigures.html

Significant Figures and Rounding Rules: a discussion of issues relating to the proper teaching of significant figures and rounding rules. This site, authored by Christopher Mulliss, includes links to other pages and to interactive tutorials. http://www.angelfire.com/oh/cmulliss/

