

# FE-Modelling of Electrical Borehole Tool Responses

Johannes B. Stoll

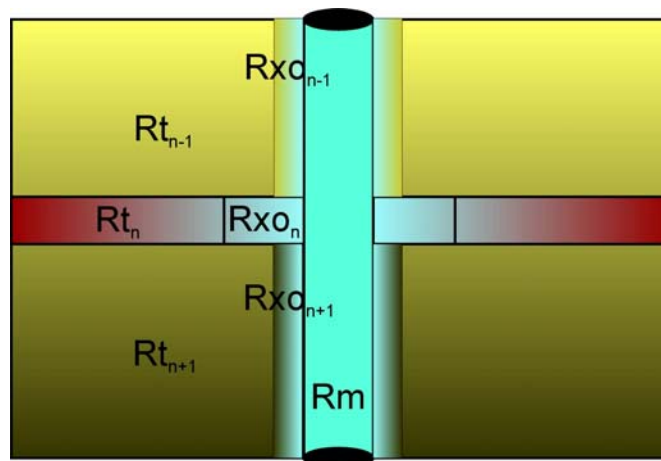
ANTARES Datensysteme GmbH, Stuhr, j.stoll@antares-geo.de

## 1 The Dual Laterolog Tool (DLT) – The Quest for $R_t$

Electric well logging was the first logging method used below ground in boreholes by the petroleum industry. Of all the rock parameters measured by logging tools, the electrical resistivity is of particular importance. Resistivity measurements are essential for determining the relative amount of hydrocarbons in a formation. In simplest terms, high resistivity indicates the possible presence of oil and gas in rock pores, since hydrocarbons are insulators. On the other hand, low resistivity indicates water, the other fluid that may be present. The parameter of greatest interest in evaluating a reservoir for its hydrocarbon content is  $R_t$ , the resistivity of a bed under consideration which has not been contaminated by borehole fluids. Logging tools measure the over-all apparent resistivity,  $R_a$ , and in order to accurately determine  $R_t$ , perturbations caused by adjacent regions must be taken into account. These regions are shown in *Figure 1* and include:

- The borehole of diameter  $d$ , filled with drilling mud of resistivity  $R_m$
- Zones encircling the borehole flushed by the borehole mud called invaded zones with resistivity  $R_{xo}$
- Adjacent layers of differing resistivity called shoulder beds, with resistivity  $R_{n-1}$  and  $R_{n+1}$ .

Because well logging is carried out with the tool immersed in the borehole mud, mud properties and borehole size can affect the accuracy of the measurement of  $R_t$ . Highly conductive mud can short-circuit currents and prevent them from penetrating deeply into a formation. Therefore it is important to accurately account for borehole effects. The effects of the borehole and adjacent beds can be decreased by designing tools to minimize their effect or by computer processing. Invasion can be resolved by using tools with several depth of investigation.



**Fig. 1** Earth model with parameters relevant to a borehole, invaded zone, lengths of invasion, and respective resistivities.

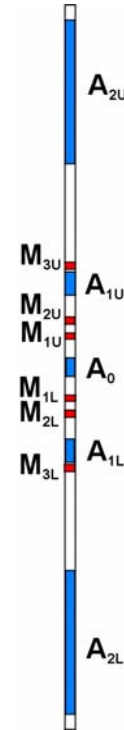
The ultimate purpose of resistivity logging is to determine hydrocarbon saturation from the true formation resistivity  $R_t$ . It has been impossible so far to design a single deep-reading measurement of  $R_t$ , which is entirely free of the effects of the borehole and invaded zone. The strategy is to design a tool with different depths of investigation. As a result, the so-

called Dual Laterolog Tool (DLT) was designed to devise a tool combination which had: (1) little borehole effect, (2) good vertical resolution and (3) two well-distributed radial depths of investigation. *Table 1* shows the dimensions of the DLT electrode configuration. It consists of two different modes, a Deep mode, with a deep depth of investigation and a Shallow mode, with a shallow depth of investigation.

**Tab. 1** Dual Laterolog Scheme

Electrode	Position	Electrode Length
$A_{3U}$	2.280	1.2m
$M_{3U}$	0.787m	0.12m
$A_{1U}$	0.680m	0.185m
$M_{2U}$	0.3683m	0.25m
$M_{1U}$	0.2413m	0.25m
$A_0$	0m	0.150m
$M_{1L}$	-0.2413m	0.25m
$M_{2L}$	-0.3683m	0.25m
$A_{1L}$	-0.680m	0.185m
$M_{3L}$	-0.787m	0.12m
$A_{3L}$	-2.280	1.2m

Positions of current and monitoring electrodes



DLT Electrode configuration

## 2 The Transfer Impedance Concept

In conductive muds, the current emitted by the current electrodes can travel inside the borehole for long distances before entering the formation. Therefore the DLT was designed to implement a current focussing to inject current laterally into the formation. The DLT is comprised of five current electrodes,  $A_0$ ,  $A_{1U}$ ,  $A_{2U}$ ,  $A_{1L}$ ,  $A_{2L}$  and six voltage electrodes,  $M_{1U}$ ,  $M_{2U}$ ,  $M_{3U}$ ,  $M_{1L}$ ,  $M_{2L}$ ,  $M_{3L}$ , called monitoring electrodes. ("U" and "L" denote upper and lower.). Electrodes  $A_{1U}$ ,  $A_{1L}$ ,  $A_{2U}$ ,  $A_{2L}$  drive currents into the formation which focus the current beam emitted by the  $A_0$  electrode. The degree of survey current focusing can be altered by varying the strength of these auxillary currents (bucking currents). In practice, the strength of the bucking currents is controlled by a feedback loop with sufficient gain to ensure that the potential gradient measured between a pair of monitoring electrodes  $M_{1U}$ ,  $M_{2L}$ ,  $M_{2U}$ ,  $M_{1L}$  is null. This condition imposes that no currents are flowing in the vertical direction in the vicinity of the monitoring electrodes and forces the survey current beam to be well focussed into any bed adjacent to the  $A_0$  electrode. Thus the  $A_0$  survey current enters the formation horizontally within the area bounded by the monitor electrodes. The monitoring conditions for DLT-Deep (LLD) are:

$$\begin{aligned}
 V_{A1U} &= V_{A1L} \\
 V_{A2U} &= V_{A2L} \\
 V_{M1U} + V_{M1L} &= V_{M2U} + V_{M2L} \\
 V_{A2U} + V_{A2L} &= V_{M3U} + V_{M3L}
 \end{aligned}$$

The apparent resistivity  $R_a$  for LLD is:  $R_a = K_{LLD} \frac{(V_{M1U} + V_{M1L})/2}{I_{A0}}$ , with  $K_{LLD}$  equal to 0.90. All

emitted currents are returned to a B electrode located at quasi infinity. The reference potential electrode, N, is located at infinity as well.

The monitoring conditions for the DLT-Shallow (LLS) are:

$$\begin{aligned} V_{A1U} &= V_{A1L} \\ V_{A2U} &= V_{A2L} \\ V_{M1U} + V_{M1L} &= V_{M2U} + V_{M2L} \\ I_{A0} + I_{A1U} + I_{A1L} + I_{A2U} + I_{A2L} &= 0 \end{aligned}$$

The apparent resistivity  $R_a$  for LLS is:  $R_a = K_{LLS} \frac{(V_{M1U} + V_{M1L})/2}{I_{A0}}$ , with  $K_{LLS}$  equal to 1.02.

These conditions must be met to compute the DLT response. There are a total of eleven electrodes, five are current electrodes and six are voltage electrodes which are supposed to be passive or connected to infinite input impedance electronics. Sixteen conditions are therefore necessary to solve for the sixteen unknowns. In order to simplify calculations, the concept of transfer impedance is introduced. If  $V_i$  is designated as the potential on electrode  $i$  with respect to infinity, and  $I_j$  as the current emitted by electrode  $j$ , then the transfer impedance  $Z_{ij}$  between electrodes  $i$  and  $j$  is defined as

$$V_{ij} = Z_{ij} I_j$$

$Z_{ij}$  can be thought of as the ratio of the voltage on electrode  $i$  to the current from electrode  $j$  which generate the voltage.

The linear system of equations describing the LLD operating conditions can be written in the matrix form as

$$\begin{bmatrix} I_{A0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & V_{A1U} & 0 & -V_{A1L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{A2U} & 0 & -V_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{M1U} & -V_{M2U} & 0 & V_{M1L} & -V_{M2L} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M3U} & 0 & 0 & -V_{M3L} \\ Z_{A0A0} I_{A0} & Z_{A0A1U} I_{A1U} & Z_{A0A2U} I_{A2U} & Z_{A0A1L} I_{A1L} & Z_{A0A2L} I_{A2L} & -V_{A0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A1U A0} I_{A0} & Z_{A1U A1U} I_{A1U} & Z_{A1U A2U} I_{A2U} & Z_{A1U A1L} I_{A1L} & Z_{A1U A2L} I_{A2L} & 0 & -V_{A1U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A2U A0} I_{A0} & Z_{A2U A1U} I_{A1U} & Z_{A2U A2U} I_{A2U} & Z_{A2U A1L} I_{A1L} & Z_{A2U A2L} I_{A2L} & 0 & 0 & -V_{A2U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A1L A0} I_{A0} & Z_{A1L A1U} I_{A1U} & Z_{A1L A2U} I_{A2U} & Z_{A1L A1L} I_{A1L} & Z_{A1L A2L} I_{A2L} & 0 & 0 & 0 & -V_{A1L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A2L A0} I_{A0} & Z_{A2L A1U} I_{A1U} & Z_{A2L A2U} I_{A2U} & Z_{A2L A1L} I_{A1L} & Z_{A2L A2L} I_{A2L} & 0 & 0 & 0 & 0 & -V_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{M1U A0} I_{A0} & Z_{M1U A1U} I_{A1U} & Z_{M1U A2U} I_{A2U} & Z_{M1U A1L} I_{A1L} & Z_{M1U A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & -V_{M1U} & 0 & 0 & 0 & 0 & 0 \\ Z_{M2U A0} I_{A0} & Z_{M2U A1U} I_{A1U} & Z_{M2U A2U} I_{A2U} & Z_{M2U A1L} I_{A1L} & Z_{M2U A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M2U} & 0 & 0 & 0 & 0 \\ Z_{M3U A0} I_{A0} & Z_{M3U A1U} I_{A1U} & Z_{M3U A2U} I_{A2U} & Z_{M3U A1L} I_{A1L} & Z_{M3U A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M3U} & 0 & 0 & 0 \\ Z_{M1L A0} I_{A0} & Z_{M1L A1U} I_{A1U} & Z_{M1L A2U} I_{A2U} & Z_{M1L A1L} I_{A1L} & Z_{M1L A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M1L} & 0 & 0 \\ Z_{M2L A0} I_{A0} & Z_{M2L A1U} I_{A1U} & Z_{M2L A2U} I_{A2U} & Z_{M2L A1L} I_{A1L} & Z_{M2L A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M2L} & 0 \\ Z_{M3L A0} I_{A0} & Z_{M3L A1U} I_{A1U} & Z_{M3L A2U} I_{A2U} & Z_{M3L A1L} I_{A1L} & Z_{M3L A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M3L} \end{bmatrix}$$

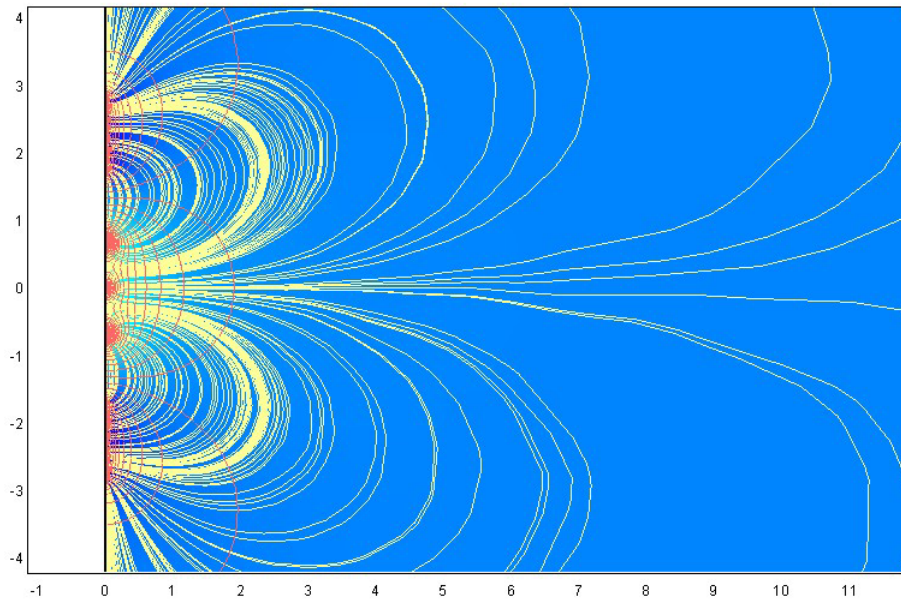
The corresponding linear system of equations describing the LLS operating conditions is:

$$\begin{bmatrix} I_{A0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & V_{A1U} & 0 & -V_{A1L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{A2U} & 0 & -V_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_{M1U} & -V_{M2U} & 0 & V_{M1L} & -V_{M2L} & 0 \\ I_{A0} & I_{A1U} & I_{A2U} & I_{A1L} & I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A0A0} I_{A0} & Z_{A0A1U} I_{A1U} & Z_{A0A2U} I_{A2U} & Z_{A0A1L} I_{A1L} & Z_{A0A2L} I_{A2L} & -V_{A0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A1U A0} I_{A0} & Z_{A1U A1U} I_{A1U} & Z_{A1U A2U} I_{A2U} & Z_{A1U A1L} I_{A1L} & Z_{A1U A2L} I_{A2L} & 0 & -V_{A1U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A2U A0} I_{A0} & Z_{A2U A1U} I_{A1U} & Z_{A2U A2U} I_{A2U} & Z_{A2U A1L} I_{A1L} & Z_{A2U A2L} I_{A2L} & 0 & 0 & -V_{A2U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A1L A0} I_{A0} & Z_{A1L A1U} I_{A1U} & Z_{A1L A2U} I_{A2U} & Z_{A1L A1L} I_{A1L} & Z_{A1L A2L} I_{A2L} & 0 & 0 & 0 & -V_{A1L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{A2L A0} I_{A0} & Z_{A2L A1U} I_{A1U} & Z_{A2L A2U} I_{A2U} & Z_{A2L A1L} I_{A1L} & Z_{A2L A2L} I_{A2L} & 0 & 0 & 0 & 0 & -V_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_{M1U A0} I_{A0} & Z_{M1U A1U} I_{A1U} & Z_{M1U A2U} I_{A2U} & Z_{M1U A1L} I_{A1L} & Z_{M1U A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & -V_{M1U} & 0 & 0 & 0 & 0 & 0 \\ Z_{M2U A0} I_{A0} & Z_{M2U A1U} I_{A1U} & Z_{M2U A2U} I_{A2U} & Z_{M2U A1L} I_{A1L} & Z_{M2U A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M2U} & 0 & 0 & 0 & 0 \\ Z_{M3U A0} I_{A0} & Z_{M3U A1U} I_{A1U} & Z_{M3U A2U} I_{A2U} & Z_{M3U A1L} I_{A1L} & Z_{M3U A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M3U} & 0 & 0 & 0 \\ Z_{M1L A0} I_{A0} & Z_{M1L A1U} I_{A1U} & Z_{M1L A2U} I_{A2U} & Z_{M1L A1L} I_{A1L} & Z_{M1L A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M1L} & 0 & 0 \\ Z_{M2L A0} I_{A0} & Z_{M2L A1U} I_{A1U} & Z_{M2L A2U} I_{A2U} & Z_{M2L A1L} I_{A1L} & Z_{M2L A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M2L} & 0 \\ Z_{M3L A0} I_{A0} & Z_{M3L A1U} I_{A1U} & Z_{M3L A2U} I_{A2U} & Z_{M3L A1L} I_{A1L} & Z_{M3L A2L} I_{A2L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V_{M3L} \end{bmatrix}$$

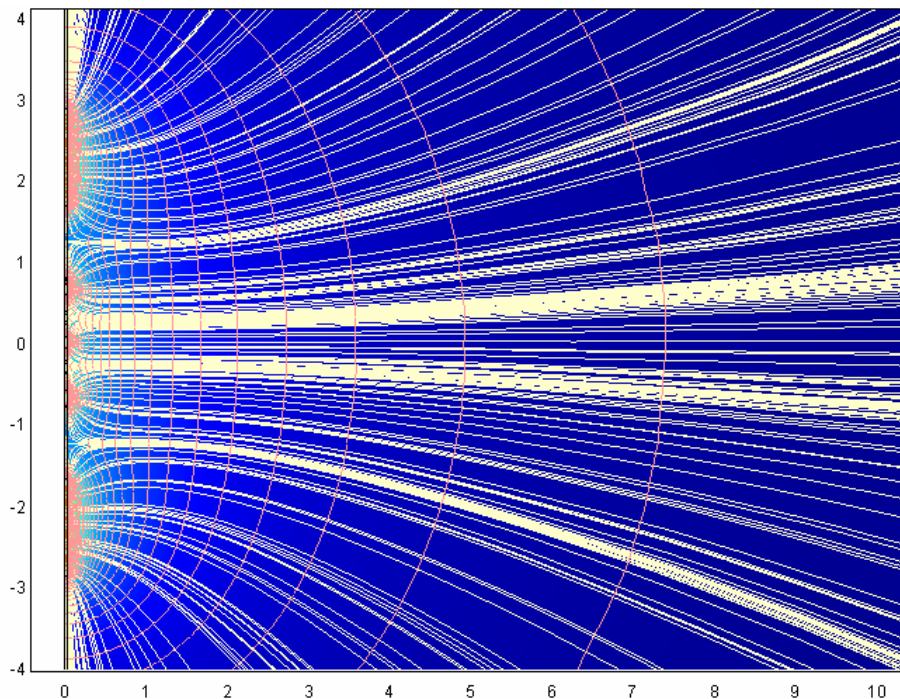
Both above linear systems are generated from two sub-systems. The first five rows of the matrix describe how the device is operated. A unit current is emitted by electrode  $A_0$  ( $I_{A0}=1$ ). There is a short circuit between electrode  $M_{1U}$  and  $M_{2L}$  ( $V_{M1U}-V_{M2L}=0$ ). There is also a short

circuit between electrode  $M_{2U}$  and  $M_{1L}$  ( $V_{M_{2U}} - V_{M_{1L}} = 0$ ). The last eleven rows of the matrix relate the potential of each electrode to the currents through their respective transfer impedance. Now, any modelling code which solves the Poisson equation, can be used to compute the transfer impedance for each current-voltage electrode pairing. The focussed tool response is calculated by solving the above system of linear equations for the unknown current ( $I_j$ ) and voltage ( $V_j$ ). The appropriate current and voltage values are then substituted in the matrix to generate the apparent resistivity read by the tool.

Figures 2 and 3 display the results of FE forward modelling for the DLL electrode configuration shown in Tab. 1. The streamline plots of the electric current for both the Laterolog Shallow (LLS) and Laterolog Deep (LLD) clearly show the focussing effect.



**Fig. 2** Streamline plot for the LLS electrode configuration



**Fig. 3** Streamline plot for the LLD electrode configuration

### 3 Radius of investigation (RI) and Borehole Correction

The contribution to the measured signal of any desired portion of the ground can be computed by using the equivalence between DC and electrostatic fields. In other words, all point sources of an electric current are replaced by point charges, so that the electrically conductive medium becomes dielectric. Each volume element of the ground will acquire electrostatic polarization due to the primary charge and acts as an electrostatic dipole, giving rise in its turn to an electrostatic potential at the measuring point. These potentials can then be integrated to give the contribution of cylindrical shells. The radial and vertical components of the electrostatic dipole moment  $\mu_r, \mu_z$  acquired by a volume element of the ground can be taken to be

$$\mu_r = \frac{1}{4\pi} \frac{\partial V}{\partial r} \cdot dr \cdot r \, d\phi dz \quad \mu_z = \frac{1}{4\pi} \frac{\partial V}{\partial z} \cdot dz \cdot r \, d\phi dr$$

The potential at one of the monitoring electrodes at point  $M_{1U}$  due to  $\mu_r, \mu_z$  of the ground element at  $(r, z)$  is

$$dV(P) = \mu_r \cdot \frac{\partial}{\partial r} [r^2 + (z - z_{M1U})^2]^{-1/2} + \mu_z \cdot \frac{\partial}{\partial z} [r^2 + (z - z_{M1U})^2]^{-1/2}$$

The contribution to the signal of a cylindrical shell of thickness  $dr$  and with the axis coincident with the centre of the tool is given by integrating  $dV(P)$  with respect to  $z$ . The radius of investigation  $RI$  is then defined by:

$$RI = \int_{z=-\infty}^{z=+\infty} dV(P)$$

Applying to finite element forward modelling the electric field for LLD and LLS is calculated stepwise for an array of vertical profiles with a spacing of 10cm. *Figure 4* depicts the integrated response of LLD and LLS. 50% of the signal read by the tool is generated within a radius of 0.35m for the LLS and 0.85m for the LLD.

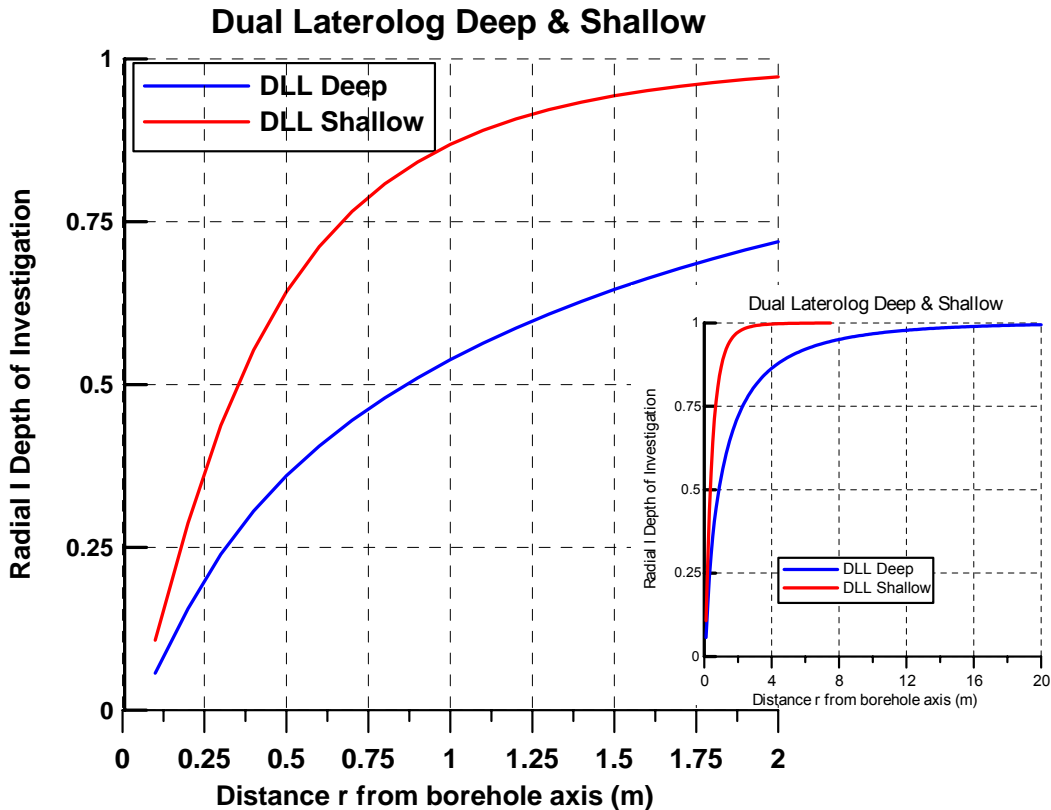
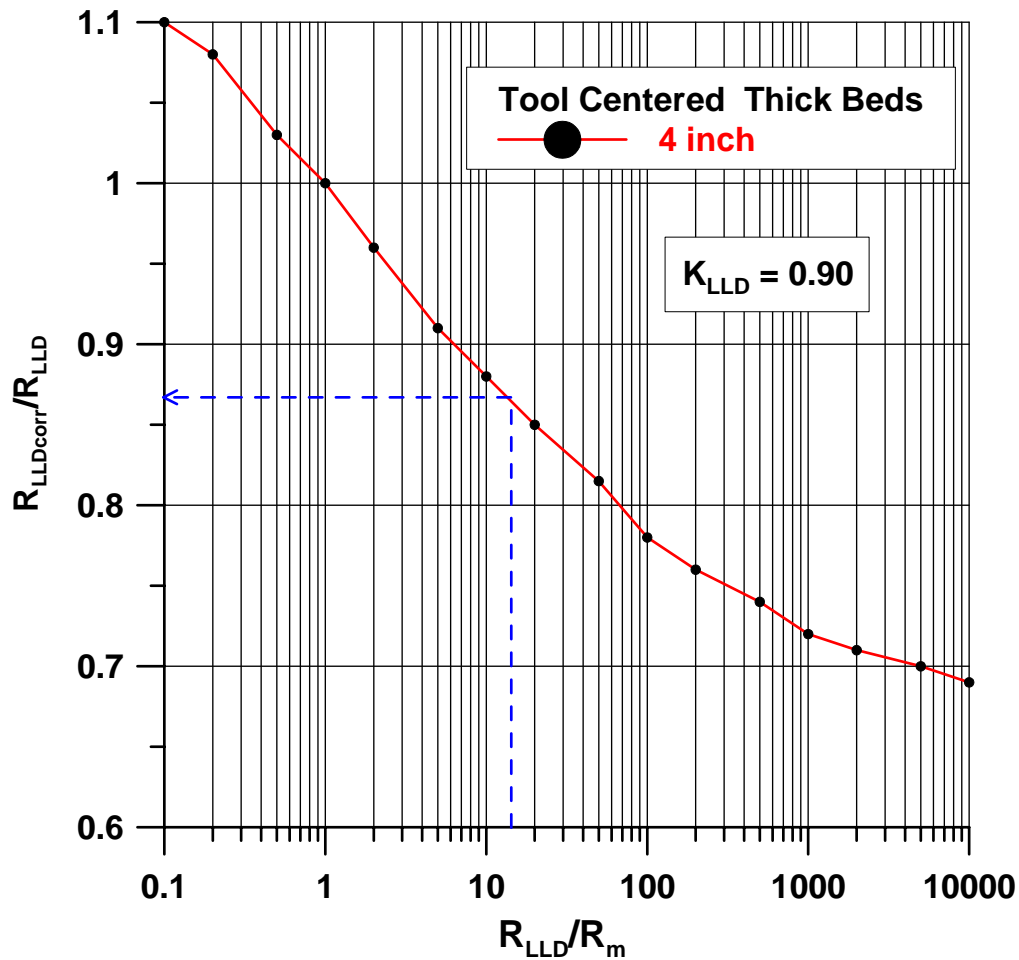


Fig. 4 Radius of investigation for the LLS (red line) and LLD (blue line).

The effect of a 4 inch borehole on the tool response  $R_{LLD}$  is modelled for a wide range for resistivity contrasts between borehole ( $R_m$ ) and formation ( $R_t$ ). For a homogeneous formation of contrast 1 (resistivity 1  $\Omega m$ )  $R_{LLD}$  is normalized to 1. For a given contrast the tool response can be corrected for the true formation resistivity  $R_t$  using the chart of *Figure 5*. For a given ratio  $R_{LLD}/R_m$  the correction factor is taken from the vertical axis.



**Fig. 5** Chart to correct the log value for the influence of the borehole. The chart is entered from the horizontal axis  $R_{LLD}/R_m$  by projecting a line upward to the borehole size curve. From that point, a line is projected to the left to derive a correction factor along the vertical axis