

Denis Prokofiev, Ph.D. and John Dunec, Ph.D.

## Multiphysics Model of Soil Phenomena Near a Well

**Abstract** The equations describing poroelastic materials are developed in detail and compared to those describing Darcy flow and structural mechanics in COMSOL Multiphysics. The difference between the coupled poroelastic equations and those in COMSOL Multiphysics is shown to be an additional source term both in Darcy and in structures. Simple verification models are run to check the accuracy and behavior of both the non-modified equations and the coupled poroelastic equations. These then form the basis for simulations of material compaction, fluid flow, and internal stresses near a wellbore using COMSOL Multiphysics. This paper shows the theoretical underpinnings of poroelastic theory. The presentation shows simulation results

**Keywords** Poroelasticity, Darcy Flow, Wellbore Analysis, COMSOL Multiphysics 3.2

---

### 1 Introduction

The theoretical development presented here of the poroelastic equations follows closely that of Wang [ref 3]. Interested readers should look to his text for an excellent, detailed presentation. The theory of Poroelasticity was first developed by Terzaghi to explain his (1D) laboratory experiments and then extended to 3D by Biot. Terzaghi elucidated the concept of effective stress. When a confined cylinder of saturated soil is suddenly compressed with an axial load, the load is initially borne entirely by the fluid. In time, however the load is shifted in part to the structural matrix. Thus the total load is held by the sum of the structural stress and the pore pressure.

---

D.Prokofiev  
Shell International Exploration and Production  
Tel: 713-245-7735  
Fax: 713-245-7599  
E-mail: Denis.Prokofiev@shell.com

J.Dunec  
COMSOL, Inc  
Tel: 650-324-9935  
Fax: 650-324-9936  
E-mail: John.Dunec@comsol.com

This important observation is called Terzaghi's Law of Effective Stresses. It is the key difference between a standard structural analysis development and a poroelastic development. This paper develops the mathematics of this phenomena and then compares the equations of poroelasticity to those in COMSOL Multiphysics. Our goal is to determine how to modify the standard equations in COMSOL to extend their applicability to poroelastic materials.

One of the difficulties faced by researchers in this field is a definitional one. Theoretical poroelasticity was concurrently developed in geomechanics, hydrogeology and petroleum engineering. These fields have different traditions in defining positive and negative values in direction, stress and strain. The reader is cautioned to check these basics for their application. In this paper, positive structural stress is a tensile stress on the structural matrix and positive pore pressure is a compressive stress on the interstitial fluid. Thus pumping fluid into a confined poroelastic solid produces both a positive pore pressure (the fluid tries to compress) and a positive structural stress (the matrix tries to expand to accommodate the fluid). If your definition of positive stress or pressure differs from this, there will be sign differences in the resulting poroelastic equations.

---

### 2 Theory – Constitutive Relations

Poroelastic theory can be developed mathematically several ways. We will follow Wang's approach and start with constitutive laws between the poroelastic variables:  $\sigma$ ,  $\varepsilon$ ,  $\zeta$  and  $p$ . For simplicity, we first consider a 1D formulation of a linear relation between the variables:

$$\varepsilon = a_{11}\sigma + a_{12}p$$

$$\zeta = a_{21}\sigma + a_{22}p$$

The poroelastic constants,  $a$ , are defined as ratios of field variables, while holding others constant on an elementary control volume. These two constitutive relations between the four variables can be arranged in a number of formulations:

Pure Compliance Formulation:

$$\varepsilon = \frac{1}{K} \sigma + \frac{\alpha}{K} p$$

$$\zeta = \frac{\alpha}{K} \sigma + \frac{\alpha}{KB} p$$

Pure Stiffness Formulation:

$$\sigma = K_u \varepsilon - (K_u B) \zeta$$

$$p = -(K_u B) \varepsilon + \frac{K_u B}{\alpha} \zeta$$

Mixed Stiffness Formulation: (the formulation used here)

$$\sigma = K \varepsilon - \alpha p$$

$$\zeta = \alpha \varepsilon + S_\varepsilon p$$

where

$$S_\varepsilon = \frac{\alpha}{K_u B}$$

is defined as the constrained specific storage coefficient and

$$K = \frac{E}{3(1-2\nu)}$$

This relates the poroelastic constant,  $K$ , to the traditional elastic constants,  $E$  and  $\nu$ .  $E$ , Youngs modulus, and  $\nu$ , Poisson's ratio, are measured under drained conditions (that is, when pore pressure is zero).

For this analysis, we are concerned with 2D, 2D axisymmetric and 3D poroelasticity. The above show the relations between 1D simplified stresses. To develop the 3D constitutive equations we first consider them in their principal coordinates. Here, by definition, the shear stresses and shear strains are zero. And the constitutive equations (in pure strain form) reduce to 4 equations for 4 dependant variables cast in terms of 4 independent variables.

$$\varepsilon_1 = \frac{1}{E} \sigma_1 - \frac{\nu}{E} \sigma_2 - \frac{\nu}{E} \sigma_3 + \frac{\alpha}{3K} p$$

$$\varepsilon_2 = -\frac{\nu}{E} \sigma_1 + \frac{1}{E} \sigma_2 - \frac{\nu}{E} \sigma_3 + \frac{\alpha}{3K} p$$

$$\varepsilon_3 = -\frac{\nu}{E} \sigma_1 - \frac{\nu}{E} \sigma_2 + \frac{1}{E} \sigma_3 + \frac{\alpha}{3K} p$$

$$\zeta = \frac{1}{3H} (\sigma_1 + \sigma_2 + \sigma_3) + \frac{p}{R}$$

Traditionally, those in the earth sciences field relate stress and strain through use of the shear modulus,  $G$ ,

rather than the Youngs modulus,  $E$ , since it is easier to measure. The two moduli are related through

$$E = 2G(1+\nu)$$

Next, generalize the principal coordinate formulation to general coordinates, using the shear modulus rather than Young's. Here we assume that changes in pore pressure will not induce shear strains. The full form of the pure compliance formulation thus becomes

$$\varepsilon_{xx} = \frac{1}{2G} \left[ \sigma_{xx} - \frac{\nu}{1+\nu} (\sigma_{11} + \sigma_{22} + \sigma_{33}) \right] + \frac{\alpha}{3K} p$$

$$\varepsilon_{yy} = \frac{1}{2G} \left[ \sigma_{yy} - \frac{\nu}{1+\nu} (\sigma_{11} + \sigma_{22} + \sigma_{33}) \right] + \frac{\alpha}{3K} p$$

$$\varepsilon_{zz} = \frac{1}{2G} \left[ \sigma_{zz} - \frac{\nu}{1+\nu} (\sigma_{11} + \sigma_{22} + \sigma_{33}) \right] + \frac{\alpha}{3K} p$$

$$\varepsilon_{xy} = \frac{1}{2G} \sigma_{xy}$$

$$\varepsilon_{yz} = \frac{1}{2G} \sigma_{yz}$$

$$\varepsilon_{xz} = \frac{1}{2G} \sigma_{xz}$$

Finally, recognizing that we will eventually need to use these constitutive equations in an elemental force balance equation:

$$-\nabla \cdot \boldsymbol{\sigma} = \mathbf{F},$$

solve the above equations for stress

$$\sigma_{xx} = 2G\varepsilon_{xx} + 2G\frac{\nu}{1-2\nu} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) - \alpha p$$

$$\sigma_{yy} = 2G\varepsilon_{yy} + 2G\frac{\nu}{1-2\nu} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) - \alpha p$$

$$\sigma_{zz} = 2G\varepsilon_{zz} + 2G\frac{\nu}{1-2\nu} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) - \alpha p$$

$$\sigma_{xy} = 2G\varepsilon_{xy}$$

$$\sigma_{xz} = 2G\varepsilon_{xz}$$

$$\sigma_{yz} = 2G\varepsilon_{yz}$$

The seven constitutive equation can also be recast in terms of volumetric strain:

$$\zeta = \alpha (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \frac{\alpha}{K_u B} p$$

### 3 Theory – Structural Relations

The six constitutive equations, in mixed stiffness form, can be substituted into the force balance

equation for an elemental volume. Assuming insignificant accelerations, and assuming our sign conventions, these are expressed as

$$\begin{aligned} -\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}\right) &= F_x \\ -\left(\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z}\right) &= F_y \\ -\left(\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right) &= F_z \end{aligned}$$

Substituting the constitutive equations into these force balance equations will lead to three equations relating the six quantities of strain to pore pressure and overall body forces. A more convenient set of unknowns is the structural displacements. The displacement is related to the strain (using indicial notation) through

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

After a quite a bit of algebra, the substitution of the constitutive equations and the strain-displacement equations into the elemental force balance equations yields our structural poroelastic equations

$$\begin{aligned} -\left[ G \nabla^2 u + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) \right] &= F_x - \alpha \frac{\partial p}{\partial x} \\ -\left[ G \nabla^2 v + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial x \partial z} \right) \right] &= F_y - \alpha \frac{\partial p}{\partial y} \\ -\left[ G \nabla^2 w + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2} \right) \right] &= F_z - \alpha \frac{\partial p}{\partial z} \end{aligned}$$

Comparing these to the COMSOL structural equations:

$$-\nabla \cdot \boldsymbol{\sigma} = \mathbf{F}$$

which, when expressed in shear moduli form, are (just the x-direction shown)

$$-\left[ G \nabla^2 u + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) \right] = F_x$$

we see that to extend these equations to account for poroelasticity, the body force term on the right side is augmented with a gradient of pressure term

$$-\nabla \cdot \boldsymbol{\sigma} = \mathbf{F} \rightarrow -\nabla \cdot \boldsymbol{\sigma} = \mathbf{F} - \alpha \nabla p$$

## 4 Theory – Flow Relations

The flow in poroelastic materials is a seepage flow between the grains of the material. Thus it is expressed as a relative velocity between fluid and solid matrix where  $\phi$  is the porosity of the material;

$$\vec{V} = \frac{\partial \mathbf{u}_{fluid}}{\partial t} - \frac{\partial \mathbf{u}_{solid}}{\partial t} = \frac{1}{\phi} \mathbf{q}$$

In defining the increment of fluid content, Biot and Willis defined this quantity as

$$\zeta = -\phi \nabla \cdot (\mathbf{u}_{fluid} - \mathbf{u}_{solid})$$

This is simply the fluid continuity equation, expressed in poroelastic terms. Differentiating in time

$$\frac{\partial \zeta}{\partial t} = -\phi \nabla \cdot \left( \frac{\partial \mathbf{u}_{fluid}}{\partial t} - \frac{\partial \mathbf{u}_{solid}}{\partial t} \right)$$

and substituting for the relative velocity we find

$$\frac{\partial \zeta}{\partial t} = -\phi \nabla \cdot \left( \frac{1}{\phi} \mathbf{q} \right) = -\nabla \cdot \mathbf{q}$$

Finally, rearranging this equation and generalizing it to include a volumetric fluid source, the continuity equation in poroelastic terms is

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \mathbf{q} = Q_s$$

Flow in porous media is frequently governed by Darcy's law which relates flow rates to gradients in pressure and elevation.

$$\mathbf{q} = -\frac{k}{\mu} \nabla (p + \rho_f g z)$$

Or, in terms of excess pressure:

$$\mathbf{q} = -\frac{k}{\mu} \nabla p_{ex}$$

Substituting this into the poroelastic flow continuity equation, above provides

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \left[ -\frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s$$

Finally, substituting the seventh constitutive relation between increment of fluid content and strain and pressure:

$$\zeta = \alpha (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \frac{\alpha}{K_u B} p$$

and the displacement-strain relations, we find

$$\left[ \alpha \frac{\partial}{\partial t} (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) + \frac{\alpha}{K_u B} \frac{\partial p}{\partial t} \right] - \nabla \cdot \left[ \frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s$$

Rearranging terms and recalling the relation between strain and deflection:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}; \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}; \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

we find

$$\left( \frac{\alpha}{K_u B} \right) \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s - \alpha \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

Finally defining the storage coefficient,

$$S_a = \frac{\alpha}{K_u B}$$

we arrive at the expression of Darcy's flow expanded to include poroelastic effects

$$S_a \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s - \alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

or its equivalent for excess pressure:

$$S_a \frac{\partial p_{ex}}{\partial t} - \nabla \cdot \left( \frac{k}{\mu} \nabla p_{ex} \right) = Q_s - \alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

As in the structural portion of the analysis, when we compare Darcy's Law in Earth Science Module version (pressure formulation):

$$S_a \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\eta} \nabla (p + \rho_f g D) \right] = Q_s$$

to the poroelastic version:

$$S_a \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s - \alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

We see that the two are identical except for an additional term (a loss term) added to the volumetric fluid source:

$$-\alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

## 5 Summary of Poroelastic Equations

The poroelastic structural equations to solve are

$$- \left[ G \nabla^2 u + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) \right] = F_x - \alpha \frac{\partial p}{\partial x}$$

$$- \left[ G \nabla^2 v + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} \right) \right] = F_y - \alpha \frac{\partial p}{\partial y}$$

$$- \left[ G \nabla^2 w + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2} \right) \right] = F_z - \alpha \frac{\partial p}{\partial z}$$

and, for flow

$$S_a \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s - \alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

These compare to the COMSOL equation in structures (here recast in terms of shear modulus):

$$- \left[ G \nabla^2 u + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) \right] = F_x$$

$$- \left[ G \nabla^2 v + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial x \partial z} \right) \right] = F_y$$

$$- \left[ G \nabla^2 w + \frac{G}{1-2\nu} \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2} \right) \right] = F_z$$

and the COMSOL equation for Darcy flow (Earth Science Module, pressure formulation):

$$S_a \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\eta} \nabla (p + \rho_f g D) \right] = Q_s$$

In both cases, the poroelastic equations and those in COMSOL multiphysics are identical with the exception of an additional source term on the right side. We need to add an additional body force term to the structural equations

$$\mathbf{F}_{COMSOL} \rightarrow \mathbf{F} - \alpha \nabla p$$

And an additional volumetric fluid source term to the Darcy equations

$$Q_{s, COMSOL} \rightarrow Q_s - \alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

Finally, the boundary conditions for the structural equations are slightly different. The stress that we are solving for is the poroelastic stress

## 6 Verification of Model – Analytic

To test the accuracy of the model we simulated several simple test cases with COMSOL Multiphysics. The first tests the structural side of the

equations and its additional term (this section) and the second tests the additional term in the Darcy flow equation (next section).

The first model is a simple unit annulus (radius = 1 meter, height = 1 meter) of Berea sandstone. The response of the sample to a unit pressure load in the well is tested against theory. Here we leave the top and bottom surfaces structurally unconstrained, though impermeable. The well bore pressure is set to unit loading and the outer curved surface is assumed to be open to the atmosphere and thus set to zero pore pressure. A small 3D model is built and meshed in COMSOL Multiphysics

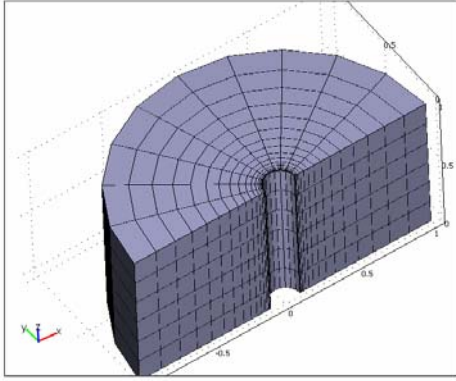


Figure 1. Geometry and Mesh

Material properties for Berea sandstone (Wang, Appendix C, & Fjaer, pg 322) are

- $G = 5.6 \text{ GPa}$  Shear Modulus
- $E = 13.1 \text{ GPa}$  Elastic Modulus
- $\nu = 0.17$  Poisson's Ratio
- $K = 6.6 \text{ GPa}$  Bulk Modulus
- $K_f = 2.3 \text{ GPa}$  Bulk Modulus of Water
- $\alpha = 0.77$  Biot-Willus Coefficient
- $\lambda = 6.75$  Drained Lamé's Modulus
- $\phi = 0.19$  Porosity
- $k = 190 \text{ mD}$  Permeability
- $B = 0.75$  Skempton's Coefficient
- $\rho = 2200 \text{ kg/m}^3$  Density

Our test annulus is simulated using these material constants. Fjaer, et al (Ref 3) provide analytic results for both the non-porous and porous cases for this geometry. These are compared to the simulation

results to verify the model. For the non-poroelastic case the radial stress near the wellbore is

$$\sigma_r = \frac{p_{out} R_{out}^2 - p_{well} R_{well}^2}{R_{out}^2 - R_{well}^2} - \frac{(p_{out} - p_{well}) R_{well}^2 R_{out}^2}{r^2 (R_{out}^2 - R_{well}^2)}$$

For the poroelastic case pore pressure can be solved analytically (Fjaer, et al, pg 118). Setting the pore pressure as the well pressure at the wellbore and the outer wall pressure at the outside curve we find

$$p = p_{out} + (p_{well} - p_{out}) \frac{\ln(r / R_{out})}{\ln(R_{well} / R_{out})}$$

Which implies a flow rate of

$$q_f = \frac{2\pi k (p_{out} - p_{well})}{\mu \ln(R_{well} / R_{out})}$$

These lead to solutions of the poroelastic equations (after much algebra) calculated by Bratli, et al, and reported in Fjaer, et al (Ref 3, pg 119) as

$$\sigma_r = \sigma_{hoopOutside} + (\sigma_{hoopOutside} - p_{well}) \frac{R_{well}^2}{R_{out}^2 - R_{well}^2} \left[ 1 - \left( \frac{R_{out}}{r} \right)^2 \right] - (p_{out} - p_{well}) \frac{1 - 2\nu}{2(1 - \nu)} \alpha \left\{ \frac{R_{well}^2}{R_{out}^2 - R_{well}^2} \left[ 1 - \left( \frac{R_{out}}{r} \right)^2 \right] + \frac{\ln(R_{out} / r)}{\ln(R_{out} / R_{well})} \right\}$$

Looking first at the non-poroelastic case, we solve for radial stress using the structural equations in the Structural Mechanics Module of COMSOL. Results are seen in figure 3.

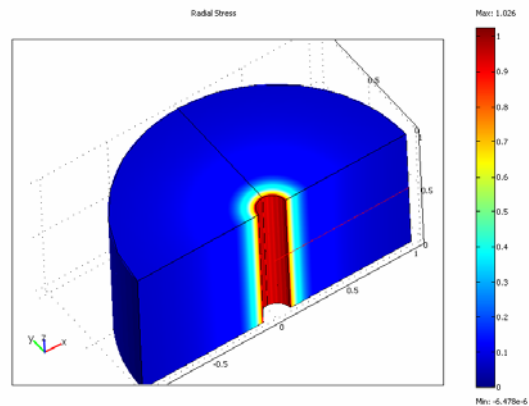


Figure 3. Radial Stress in Sample

Comparing this to the theoretical results from Fjaer, et al, we find excellent agreement

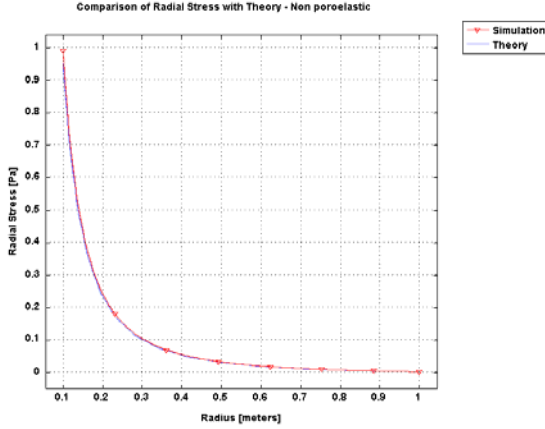


Figure 3. Comparison of theoretical radial stress to COMSOL computation

Looking at the result as a percentage error we find the accuracy to be roughly under 5%, despite a relatively coarse mesh. The percent accuracy degrades artificially near the outside wall because the stress goes to zero here.

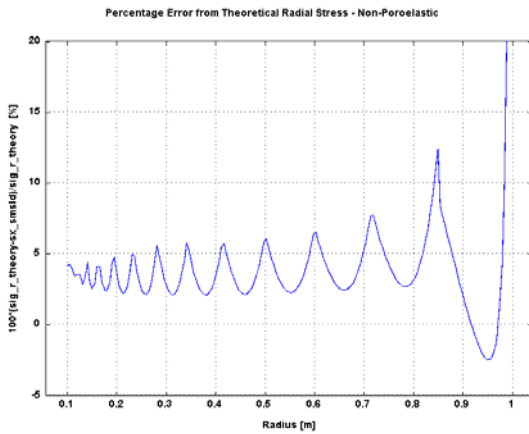


Figure 4. Percentage Error: Simulation to Theory

## 7 Verification of Model – Mandel Cryer

The second overall test is to duplicate the character of the solution of Mandel's problem. In this problem an infinitely long rectangular region is compressed between two impermeable, stiff plates. A suddenly, applied compressive load is applied between the plates at time zero.

This results in a non-monotonic pore pressure vs time curve for points inside the porous domain. The pore pressure rises in response to supporting the suddenly applied load and then decays as the fluid is pushed out the sides and the load is transferred to the solid matrix. In particular, the non-monotonic behavior arises from the additional source term in Darcy's law.

$$S_{\alpha} \frac{\partial p}{\partial t} - \nabla \cdot \left[ \frac{k}{\mu} \nabla (p + \rho_f g z) \right] = Q_s - \alpha \frac{\partial}{\partial t} (\nabla \cdot \mathbf{u})$$

We can see its effect by running two simulations, the first with the time derivative of the dilation and the second without. Mandel found that without this term the pressure vs time curve monotonically decreases. Whereas if this term is included, the pressure vs time curve is not monotonic but rather rises and then falls monotonically.

To test this behavior in COMSOL Multiphysics, we simplified the test case even further. Here we simulated an infinite cylinder of sandstone. At time equals zero, we apply a sudden compressive radial load to the entire outside of the cylinder. The liquid is allowed to escape from this same outer wall.

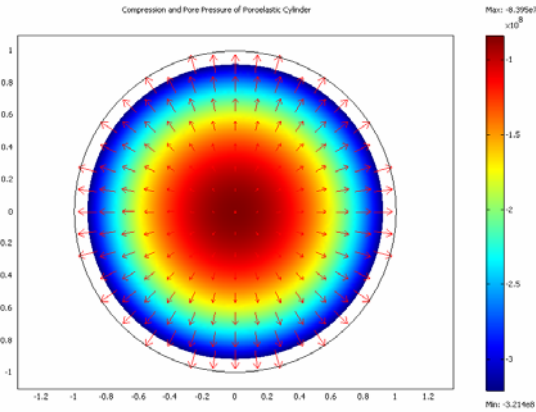


Figure 5. Mandel-Cryer Compaction

Comparing pressure vs time plots, with and without the extra source term in the Darcy equation for pore pressure measured at the center of the cylinder we see the expected behaviour.

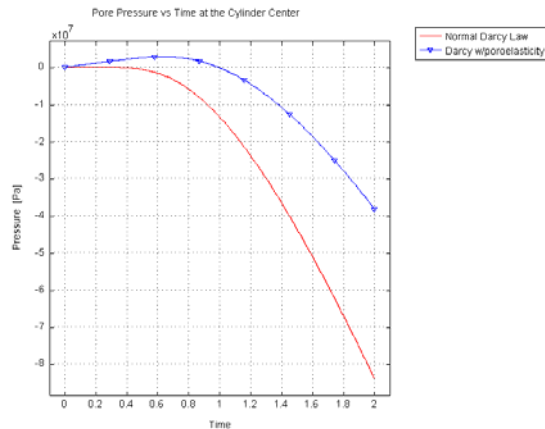


Figure 6. Comparing effect of extra Darcy term

These tests provide confidence in both the additional mathematical terms to represent poroelastic effect properly and that we have the correct signs (additive or subtractive) for them.

These modified equations form the basis of the simulation results presented at the Boston Conference.

**Acknowledgements** Thanks to Dr. Jan Wang, Prof. Tapan Mukerji, and Dr. Leigh Soutter for considerable advice and insights throughout this study.

---

#### References

1. Fjaer, E., Holt, R., Horsrud, P., Risenen, R., Petroleum Related Rock Mechanics, pp 111-113, 117-119, Elsevier, New York (1992)
2. Fung, Y.C., Foundations of Solid Mechanics, pp 392-393, Prentice-Hall, Englewood Cliffs, New Jersey (1965)
3. Wang, H.F., Theory of Linear Poroelasticity, pp 10-86. Princeton University Press, Princeton, New Jersey (2000)