Comparison of Direct and Iterative Solvers for Finite-Difference GPU-Based Valuation of American Options

Ilya Pogorelov, Paul Mullowney, Peter Messmer
(Tech-X Corporation)
Andrey Itkin and Lewis Biscamp
(Chicago Trading Company)

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Introduction and Motivation

- Modern option trading systems require highly efficient and accurate algorithms for computing theoretical values and Greeks of American options
- This requirement translates into solving concurrently multiple PDEs under very challenging time and hardware constraints
- Recent advances in GPU computing allowed us to address this problem in a highly efficient and cost-effective way
- We present results for one direct (cyclic reduction) and one iterative (biconjugate gradient) solver for the banded linear system resulting from discretization of the Black-Scholes-Merton equation

Finite Difference-Based Pricing

- Assume the underlying asset price *S(t)* undergoes geometric Brownian motion

$$dS(t) = S(t)[(r - \delta)dt + \sigma dW]$$

- American options can be exercised at a previously agreed-upon "strike price" *K*, at any time prior to maturity *T* (discrete simulation/exercise times *t = 0, …, T* in FD models)
- Theoretical value *f* of a put option written on a non-dividend-paying stock satisfies a parabolic PDE (Black-Scholes-Merton):

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

- Solving backwards in time, the "initial" condition at *t = T* is *f(t,S) = max(0, K-S)* for both European and American puts

FD-Based Pricing (continued)

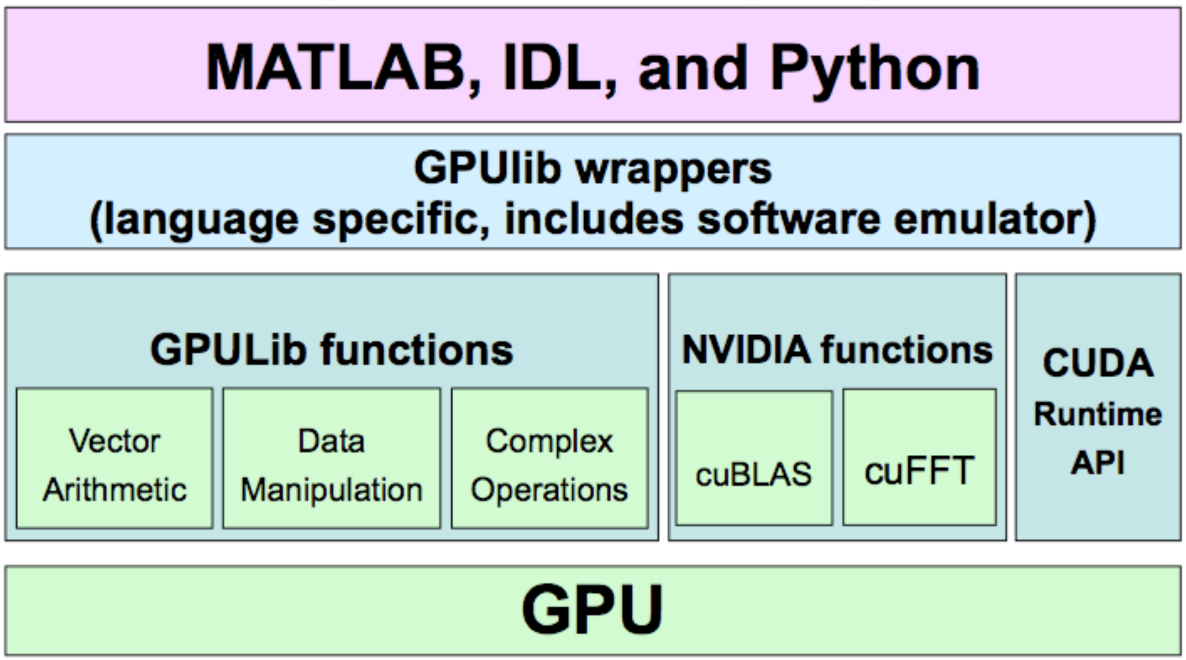
- Domain boundaries are chosen at $S = S_{max}$ such that $f(t,S_{max}) = 0$, and at $S = S_{min} = 0$ where $f(t,S_{min}) = K$ for an American put and $f(t,S_{min}) = K\ exp(-r(T-t))$ for a European put
- Implicit discretization schemes result in band-diagonal system solves at every timestep
- Guaranteed stability
- We studied performance on a GPU of cyclic reduction and biconjugate gradient solvers for the tridiagonal system resulting from an $O(\Delta s^2,\Delta t)$ discretization scheme
- A very large number of systems have to be solved simultaneously, each for a different choice of the option parameters K, T, r, σ

Problem Description

- In practice, a very large number of linear systems result from a parameter space "sweep"
- For each underlying name, a (small) number of maturities *T* and multiple strike prices *K* are considered
- Varying *K* modifies initial and boundary conditions, but not the matrix
- In addition, computation of Greeks requires re-computing the theoretical values of the options with new values of *r* and *σ*
- This yields *N* concurrent linear system solves per timestep, where *N* is the number of points in parameter space
- A task-farming-based parallelism, ideally suited for GPU-based computing

Tech-X Corporation's GPULib

- GPULib: an easy-to-use software library for computation acceleration using Graphics Processing Units (GPUs)
- GPULib provides a large collection of GPU vector operations in Very High Level languages



GPULib (cont-d)

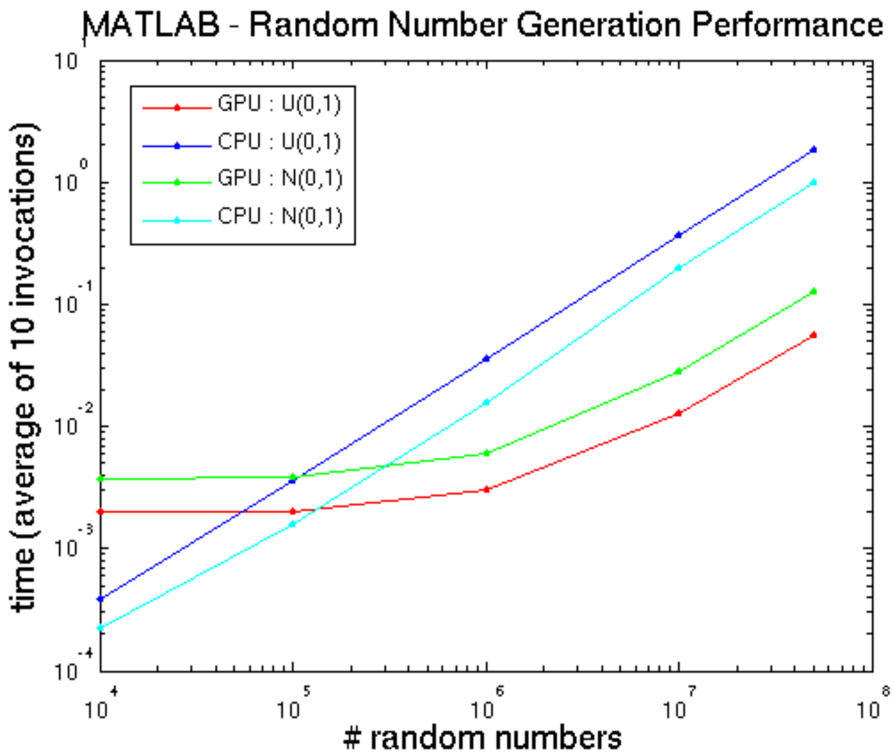
- GPU viewed as co-processor
- Explicit data transfer to/from GPU
- Interface to GPU matches language style
- Performance via vector operations on GPU
- No need for low-level code development
- Supports CUDA enabled devices

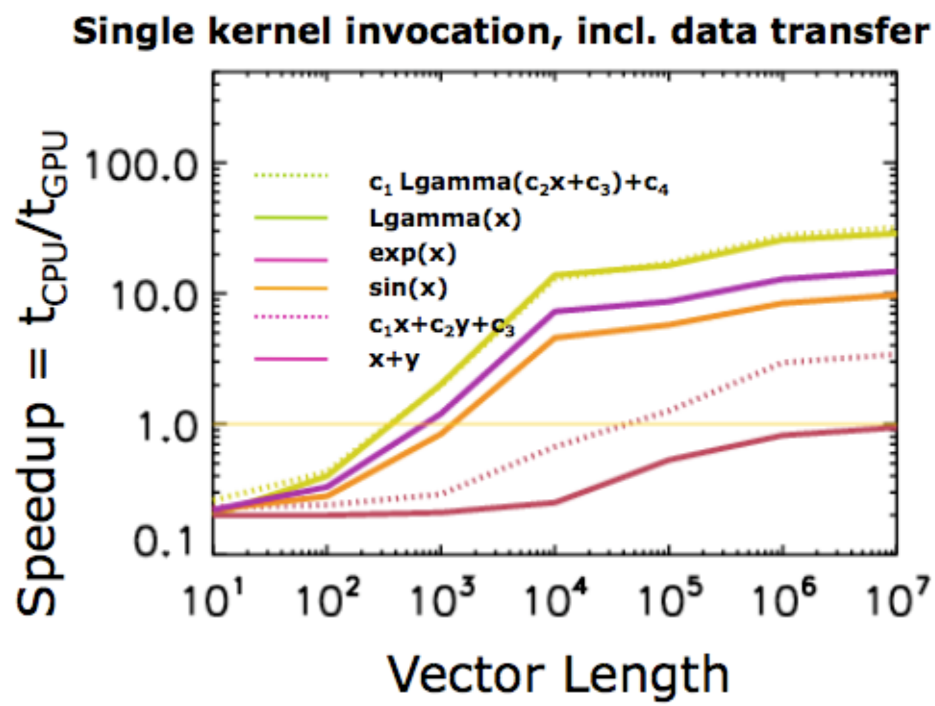
GPULib (cont-d)

- In addition to cuBLAS and cuFFT, GPULib has a broad set of:
  - Vector arithmetic operations
  - Reductions (e.g., dot products)
  - Interpolation operations
  - Array reshaping
  - Advanced physics algorithms
  - Mersenne Twister RNG, …

Messmer, Mullowney, and Granger, "GPULib: GPU Computing in High Level Languages", Computing in Science and Engineering, 10(5), 70-73 (2008)

Vector Kernel Performance





CPU: Intel Core2 6400 (dual core), 2.1 GHz, 3 GB RAM , GPU: NVIDIA GeForce 8800 GTX (128 processing elements)

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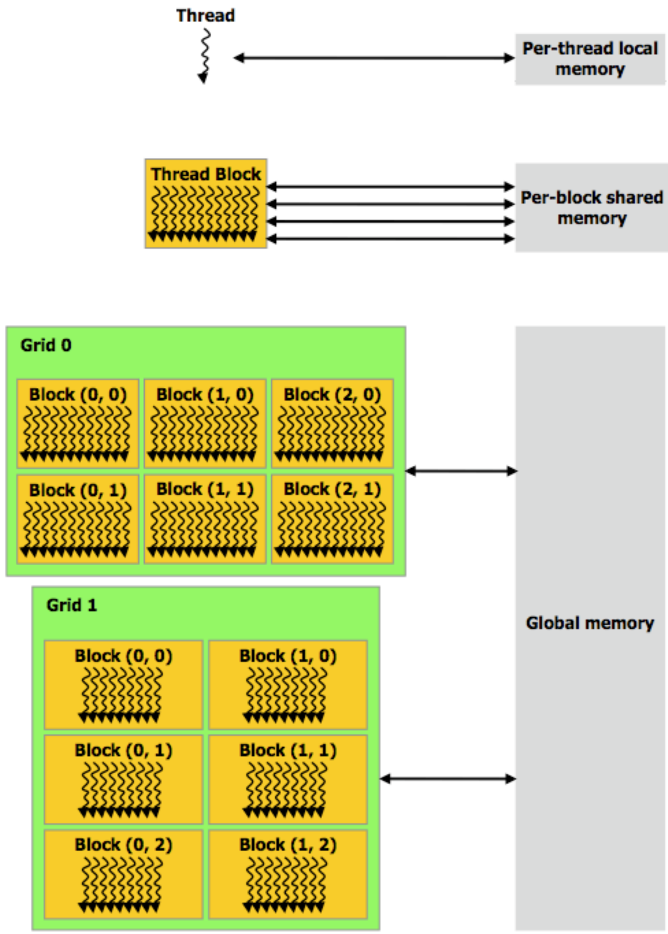
Implementing Linear Solvers on GPUs



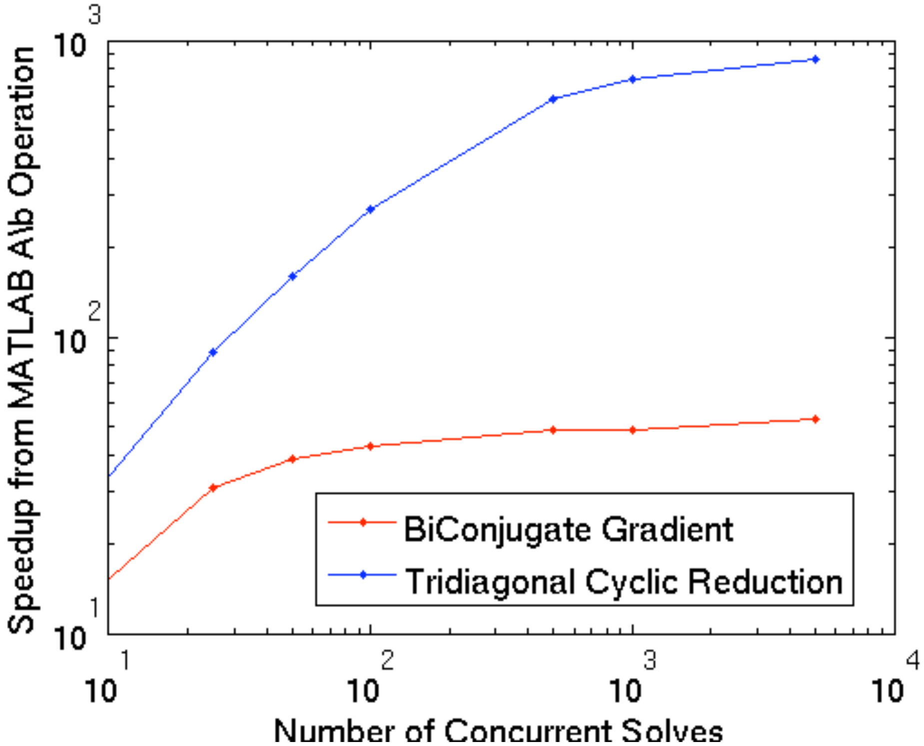
GPU Memory (From "NVIDIA CUDA Programming Guide")

- Test system: NVIDIA TESLA C1060 GPU
- Computation strategy: one thread-block per linear system solve
- Each GPU thread-block has 256 threads and 16k shared memory
- Systems are limited to 256 equation so as to keep temporary data in shared memory (both CR and BiCG)
- Each thread effectively handles one 'row' in the vector arithmetic

Implementing Linear Solvers on GPUs (continued)

- We implemented the cyclic reduction and bicongugate gradient algorithms
- Algorithms are implemented in single precision on the GPU
- One of the main issues: efficient reduction operations (e.g., dot product) in BiCG require many thread synchronization steps
- Several reduction operations *per iteration*
- Reductions are likely the main source of slowdown in BiCG

Algorithm Performance



The speedup of the GPULib cyclic reduction and biconjugate gradient solvers vs MATLAB's A\b solve. NVIDIA TESLA C1060 GPU and 2.4 GHz Intel Core 2 Duo used for these tests

- Test system: NVIDIA TESLA C1060 GPU
- Up to N = 5000 systems (points in parameter space in our sweeps)
- We expect comparable results for parameter sweeps with different matrices
- Figure to the left shows the speedup of the GPULib cyclic reduction and biconjugate gradient solvers versus MATLAB's A\b solve on the 2.4GHz Intel Core 2 Duo CPU
- CR shows 1000X and BiCG shows 50X acceleration for N = 5000

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