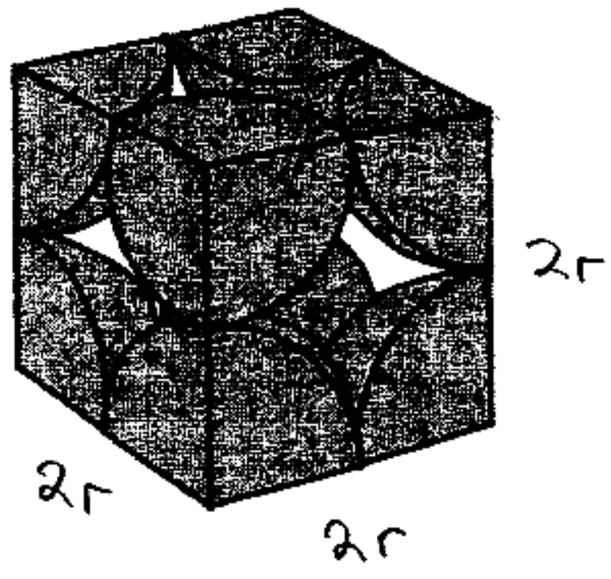
SIMPLE CUBIC:



ALL SIDES = $2r$

VOLUME = $(2r)^3 = 8r^3$

1 ATOM PER UNIT CELL

PACKING EFFICIENCY:

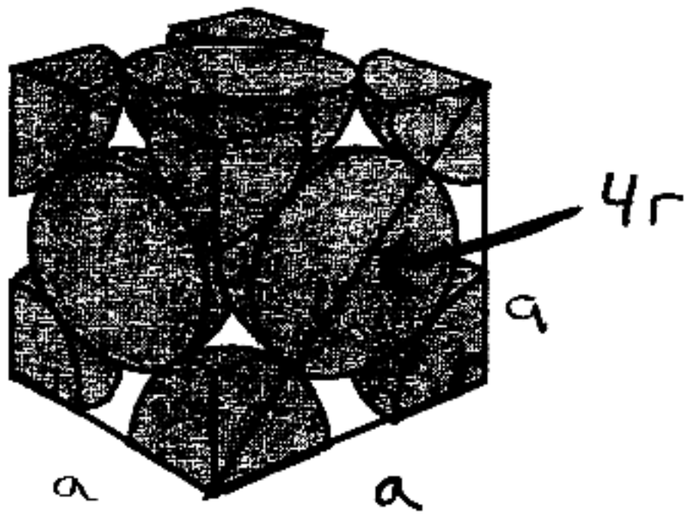
$$\frac{\text{VOLUME OF ATOM(S)}}{\text{VOLUME OF UNIT CELL}} \times 100 = \text{P.E.}$$

FOR SIMPLE CUBIC:

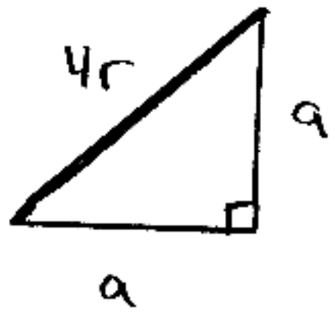
(VOLUME OF ONE ATOM = $\frac{4}{3}\pi r^3$)

$$\frac{\frac{4}{3}\pi r^3}{8r^3} \times 100 = \frac{\frac{4}{3}\pi}{8} \times 100 = 52.36\,\%$$

FACE CENTERED CUBIC:



SIDES



$$a^2 + a^2 = (4r)^2$$

$$2a^2 = 16r^2$$

$$a^2 = 8r^2$$

$$a = 2\sqrt{2}\,r$$ ← ALL SIDES HAVE THIS LENGTH

4 ATOMS PER UNIT CELL

$$\text{VOLUME} = (2\sqrt{2}\,r)^3 = 8(\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2})r^3$$

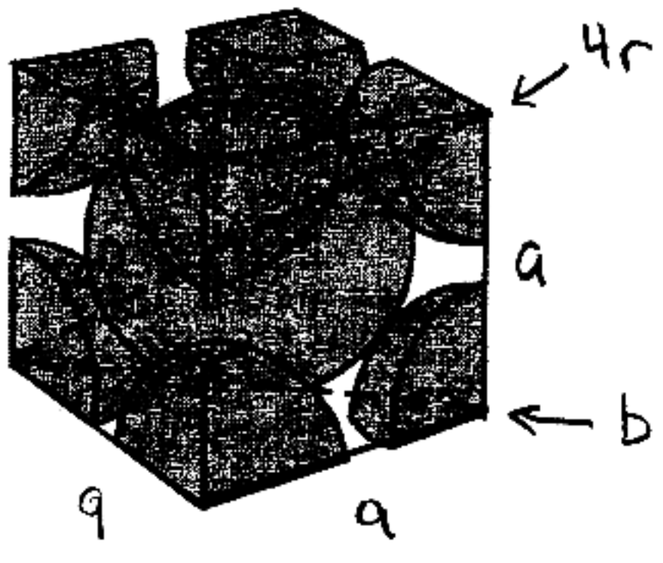
$$= 8(\sqrt{8})r^3$$

$$= 16\sqrt{2}\,r^3$$

PACKING EFFICIENCY:

$$\frac{4\left(\frac{4}{3}\pi r^3\right)}{16\sqrt{2}\,r^3} \times 100 = \frac{\pi}{3\sqrt{2}} \times 100 = 74.05\%$$

BODY CENTERED CUBIC:

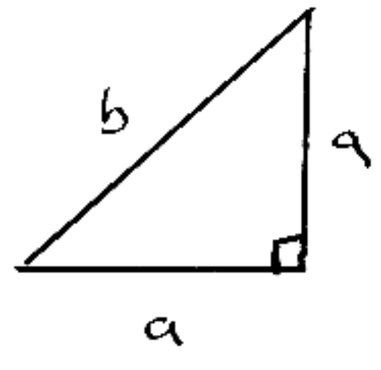


2 ATOMS PER UNIT CELL

$$b^2 = a^2 + a^2$$

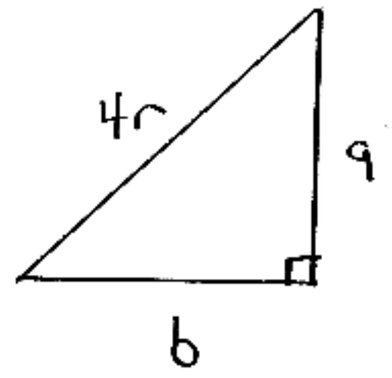
$$b^2 = 2a^2$$

$$b = \sqrt{2}a$$



$$(4r)^2 = a^2 + b^2$$

$$\downarrow \quad b = \sqrt{2}a$$



$$16r^2 = a^2 + 2a^2$$

$$16r^2 = 3a^2$$

$$\frac{16r^2}{3} = a^2$$

$$a = \frac{\sqrt{16}\,r}{\sqrt{3}} = \frac{4r}{\sqrt{3}}$$ ← ALL SIDES HAVE THIS LENGTH

$$\text{VOLUME} = \left(\frac{4}{\sqrt{3}}r\right)^3 = \frac{64r^3}{\sqrt{27}} = \frac{64r^3}{3\sqrt{3}}$$

PACKING EFFICIENCY:

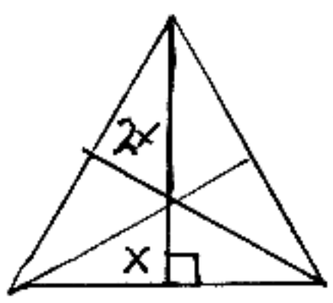
$$\frac{2\left(\frac{4}{3}\pi r^3\right)}{\frac{64r^3}{3\sqrt{3}}} \times 100 = 68.02\%$$

HEXAGANOL:

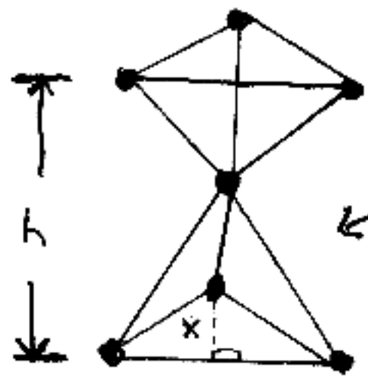
6 ATOMS SHOWN

$\frac{6}{3}$ = 2 ATOMS PER UNIT CELL

HEIGHT OF PYRAMID:

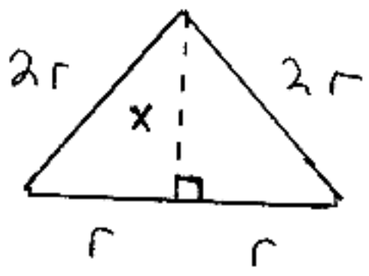


TO GET h:



ALL DISTANCES ARE $2r$

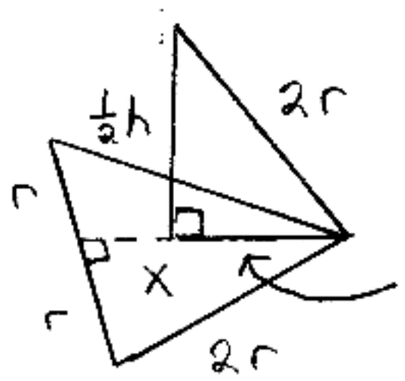
BOTTOM TRIANGLE



$x^2 + r^2 = (2r)^2$

$x^2 = 3r^2$

$x = \sqrt{3}r$



$\frac{2}{3}x = \frac{2\sqrt{3}}{3}r$

$\left(\frac{1}{2}h\right)^2 + \left(\frac{2\sqrt{3}}{3}r\right)^2 = (2r)^2$

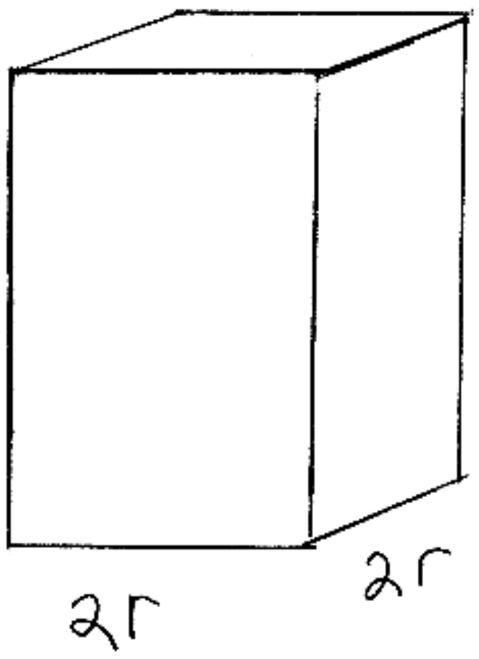
$\left(\frac{1}{2}h\right)^2 = 4r^2 - \frac{4 \cdot 3}{9}r^2 = 4r^2 - \frac{4}{3}r^2$

$\left(\frac{1}{2}h\right)^2 = 2\frac{2}{3}r^2$

$\frac{1}{2}h = \sqrt{2\frac{2}{3}}r$

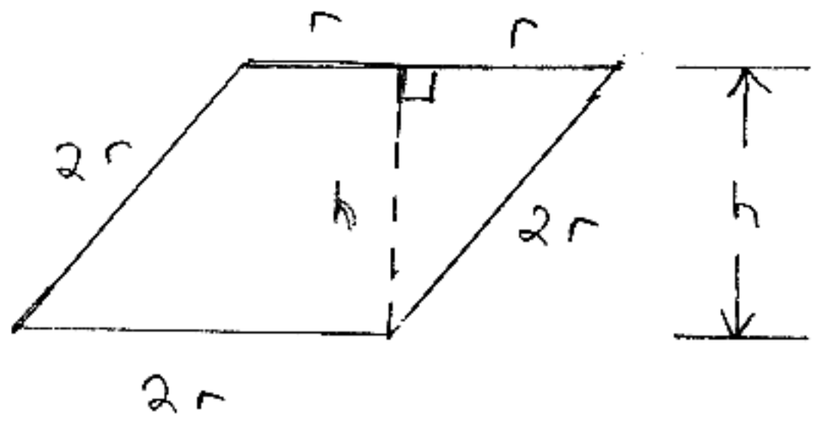
$h = 2\sqrt{2\frac{2}{3}}r$

VOLUME OF UNIT CELL:



$2\sqrt{2\frac{2}{3}}\,r$

VOLUME = AREA OF BASE × HEIGHT



AREA = LENGTH OF BASE × HEIGHT

$h^2 + r^2 = (2r)^2$

$h^2 = 4r^2 - r^2 = 3r^2$

$h = \sqrt{3}\,r$

AREA $= (2r)(\sqrt{3}\,r) = 2\sqrt{3}\,r^2$

VOLUME $= \left(2\sqrt{3}r^2\right)\left(2\sqrt{2\frac{2}{3}}\,r\right)$

VOLUME $= 4\sqrt{8}\,r^3 = 8\sqrt{2}\,r^3$

PACKING EFFICIENCY: (2 ATOMS / UNIT CELL)

$$\frac{2\left(\frac{4}{3}\pi r^3\right)}{8\sqrt{2}\,r^3} \times 100 = 74.05\%$$